Engineering Notes

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Optimum Data Utilization for Parameter Identification with Application to Lifting Rotors

D. Banerjee* and K. Hohenemser†
Washington University, St. Louis, Mo.

Introduction

Methods for state and parameter estimation from transients are widely used in aircraft testing.[1,2] These methods look promising also for wind tunnel testing and may well drastically reduce the amount of test efforts as compared to, e.g., frequency response testing. The following computer simulation study was performed in preparation of transient wind tunnel tests with a lifting rotor model. There are several parameter estimation methods applicable to transient test results. We found best suited for our case the so called maximum likelihood method which, in the absence of modeling errors and input noise, reduces to what is also called the output error method. The principal advantage of the maximum likelihood method is that it produces good estimates of the parameter error covariances in the form of the Cramér-Rao lower bounds for these covariances without needing a priori estimates of the parameter covariances.

In transient testing, the question arises what kind of input should be selected. In some recent studies, criteria were proposed to define an optimum input. They are not well-suited for our problem. Instead, we pose here the question: Given an input function, what is the minimal quantity of measured data to achieve the best possible accuracy of the identified parameters? If the data length is too short, the parameter errors will be larger than necessary. If the data length is too long, an unnecessary amount of data must be processed without an improvement in the parameter accuracy.

Two Proposals for Optimal Input Design

Analytic solutions of the problem of optimal input design require the minimization of a cost function. Stepner and Mehra[1] use the sensitivity of the system response to the unknown parameters as the performance criterion for optimal input design. The time of the transient is assumed to be fixed. The measurement equation is

$$y_m(t) = y(x,\theta,u,t) + v(t) \tag{1}$$

x is the state vector, θ the parameter vector, u the input vector, $v(t)$ the additive measurement noise with covariance matrix R. The scalar performance index selected in Ref. 1 is

$$J = \text{Trace}\,(WM) \tag{2}$$

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*Research Assistant, Dept. of Mechanical Engineering.
†Professor Emeritus, Dept. of Mechanical Engineering. Associate Fellow AIAA.

where

$$M = \int_{t_0}^{t_f} (\partial y/\partial\theta)^T R^{-1} (\partial y/\partial\theta)\,dt \tag{3}$$

The weighting matrix W is based on the relative importance of the parameter accuracies. The cost criterion J is optimized using the "energy constraint"

$$E = \int_{t_0}^{t_f} u^T u\,dt \tag{4}$$

The optimum input u can be determined as a two-point boundary value problem. It should be noted that the "energy constraint" (4) has no physical significance but is a convenient device to obtain smooth input functions. Physically, the input will usually be limited by amplitude rather than by the quadratic criterion (4) and quite different "optimal" inputs can be expected.

Chen[3] attacks the problem of optimal input design in an entirely different way as a time-optimal control problem by minimizing

$$J = \int_{t_0}^{t_f} dt \tag{5}$$

under a set of constraints including the system equations, information matrix equation, and a magnitude constraint of the input. One can show that for linear input $u(t)$ into the system equation and for an input matrix independent of any unknown parameter, the optimal input is of the "bang-bang" form between the amplitude constraints. The solution of this problem requires a computer search, which was not performed in Ref. 3. Rather an arbituary set of bang-bang inputs in the form of Walsh functions was shown to result in a specific case in lower values of the Cramér-Rao lower bound of parameter covariances than those obtained by using Mehra's "optimal input".

Although the input amplitude constraint used by Chen is physically more significant than the quadratic constraint (4) used by Mehra, the actual constraints are usually still more complex. In cases of airplanes or lifting rotors, one usually wishes to limit the response to the linear substall regime, since the analytic model to be identified is often a linear one. The stall boundary is, however, a complex function of the input and cannot be represented by an amplitude constraint for the input transient. This is particularly relevant for the lifting rotor, so that neither the Mehra nor the Chen input optimization criteria are useful for lifting rotor applications.

Optimal Data Utilization for Given Input Function

The Cramér-Rao lower bound has been defined only for a vector of sampled measurements and not for the continuous case.[2,4] For high sampling rate, one can define an approximate differential equation for the information matrix in the following way: set

$$S_i \triangleq (\partial v/\partial\theta)_i \tag{6}$$

where

$$v = y_m \hat{y}, \qquad \text{see Eq.(1)}$$

where $\hat{y}$ is the estimate of y, then

$$M=\sum_{i=1}^{N} S_i^T R^{-1} S_i = \frac{1}{\Delta t}\sum_{i=1}^{N} S_i^T R^{-1} S_i \, \Delta t$$

$$\approx (1/\Delta t)\int_{t_0}^{t_f=N\Delta t} S^T R^{-1} S \, dt \qquad (7)$$

M^{-1} is the Cramér-Rao lower bound for the parameter covariance matrix. Taking the derivative of M^{-1} with respect to t_f

$$\mathrm{d}M^{-1}/\mathrm{d}t_f = M^{-1}(\tfrac{1}{3}M/\partial t_f)M^{-1} =$$

$$-(1/\Delta t)M^{-1} S^T R^{-1} S M^{-1} \qquad (8)$$

We can now use the approximately valid differential equation (8) to obtain some insight into ways of best data utilization. Since it is impractical to use for the integration of Eq. (8) infinity as initial condition, it is recommended to determine M^{-1} for a small time period, say for $N=10$, from Eq. (7) and integrate Eq. (8) with the solution to Eq. (7) as initial conditions. Since S includes parameter estimates, one needs a preliminary estimation of the unknown parameters in order to use Eq. (8).

Application to a Case of Lifting Rotor Parameter Identification

Using the simplest analytic model of a lifting rotor, a straight blade flapping about the rotor center, one has in a rotating frame of reference for the flapping angle β the following equation[5]

$$\ddot{\beta}+(\gamma/2)C(t)\dot{\beta}+[P^2+(\gamma/2)K(t)]\beta=(\gamma/2)m_\theta(t)\theta \qquad (9)$$

where γ is the blade Lock number, P is the blade flapping natural frequency in the rotating system, θ is the instantaneous blade pitch angle, and C, K, m_θ are periodic functions of time defined in Ref. 5. One rotor revolution corresponds to $t=2\pi$. As a first approximation of dynamic rotor wake effects, one can use, instead of the actual blade. Lock number, an equivalent smaller value of γ that is treated here as an unknown. In addition, the collective pitch angle θ_0, due to airfoil inaccuracies and pitch setting errors, will also not be well-known. The problem then is to determine from blade flapping transients caused by blade pitch inputs, the equivalent Lock number γ and the equivalent collective pitch setting θ_0.

The lifting rotor wind tunnel model described in Ref. 6 allows excitation of progressing and regressing flapping modes at various frequencies. By a minor modification of this model, progressing or regressing transients can be excited. One can describe such inputs as pitch stirring transients. In a helicopter, this would amount to cyclic stick stirring, whereby the amplitude of the cyclic pitch would remain constant while the frequency of the stirring motion changes. The blade pitch input for such a stirring transient is selected to satisfy the equation:

$$\theta = 1.5\cos[\omega(t-t_0)+t]+\theta_0 \qquad (10)$$

where ω is the angular stirring speed in the sense of blade rotation

$$\omega=\begin{cases} 0 & o\le t<t_0 \\ -(0.1/\pi)(t-t_0) & t_0\le t\le t_f, \text{ where } t_0=12 \end{cases} \qquad (11)$$

Test results for the blade flapping transients will be presented at a later time when they become available. Here we are concerned with the problem of designing the tests in such a way that the test data will be sufficient to determine the two unknown parameters γ and θ_0 with good accuracy.

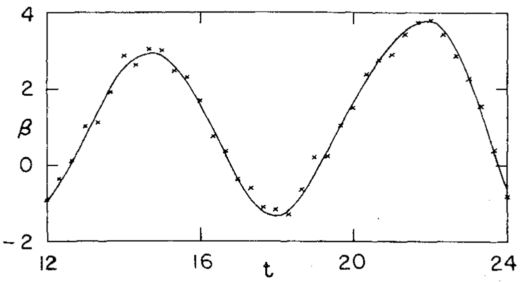

Fig. 1 Simulated flapping measurements.

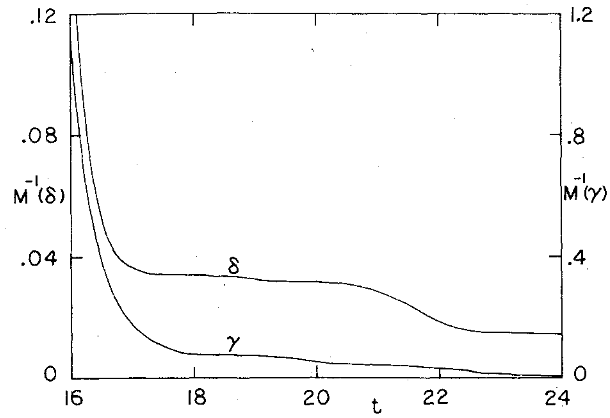

Fig. 2 Cramér-Rao lower bounds M^{-1} (γ) and M^{-1} (δ) vs time.

The simulated identification analysis was performed under the assumption of a random zero mean white noise sequence superimposed on the analytic flapping transient. The analytical transient was obtained for θ_0 = 2 deg, advance ratio $\mu=0.4$, and $\gamma=5.0$. For convenience, the parameters $\delta=\gamma\theta_0$ and γ instead of θ_0 and γ were identified.

Figure 1 shows the estimated flapping response together with the simulated measurement data (crosses). Pitch stirring is initiated at $t=12$. Figure 2 shows M^{-1} (γ) and $M^{-1}(\delta)$ from Eq. (8) between $t=16$ and $t=24$ for identified parameter values of $\gamma=4.91$, $\delta=9.83$. The curves in Fig. 2 were found to be insensitive to parameter inaccuracies. Note the steep descent of the curves to about $t=17.5$. It would, therefore, be unacceptable to use the data up to less than the time $t=17.5$. However, there is a second descent to $t=23.0$ causing another improvement. No further descents occur beyond $t=24$, and this region has been omitted in Fig. 2. It is clear that the selection of $t=24.0$ is a good one, that the use of fewer data would result in a substantial decrease in parameter accuracy, and that the use of additional data is unnecessary. A number of identification runs using different data lengths resulted in parameter accuracies that conform with the parameter covariances shown in Fig. 2.

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Separation Ahead of Steps on Swept Wings

Louis G. Kaufman II*
Grumman Aerospace Corporation, Bethpage, N. Y.

and

L. Michael Freeman†
USAF Flight Dynamics Laboratory, Wright-Patterson, Air Force Base, Ohio

Introduction

THE character of the boundary layer is one of the most important factors influencing separation; laminar boundary layers separate more readily and more extensively than turbulent boundary layers.[1,2] Many investigations have been made of two-dimensional laminar, transitional or turbulent boundary-layer separation, but few have examined the effects of boundary-layer transition on three-dimensional separated flows. Korkegi[3] described the interaction of a planar swept shock wave with the boundary layer on an unswept flat plate. The present work addresses the complementary problem of a shock caused by an unswept obstacle interacting with the boundary layer on a swept flat plate.

Theoretical Analysis

Whitehead and Keyes[4], Stollery[5] and others have observed "dumbbell" shaped regions of separated flow ahead of trailing edge flaps on delta wings, such as that sketched in Fig. 1. The shape of this region, over half the planform, reminds one of plots showing the variation of separation length, ℓ, with Reynolds number (e.g., Ref. 4). For small Reynolds numbers the boundary layer remains laminar through reattachment, and the extent of separation increases with increasing Reynolds number. This trend is reversed (for larger Reynolds number, ℓ decreases with increasing Reynolds number) when transition occurs in the separated shear layer prior to reattachment. At still larger Reynolds numbers, transition occurs upstream of separation, and the extent of separation becomes insensitive to further increases in Reynolds number.

Boundary-layer transition on sharp-leading-edge swept wings at zero angle of attack occurs in a region parallel to the wing leading edge, and, in the absence of protuberances or other disturbances, flow over the wing remains very nearly streamwise.[4] Thus, considering a strip-type analysis, turbulent separation could occur inboard while outboard the separation could be laminar. In between, the separated flow would be characteristically transitional (cf., Fig. 2.).

Although the motivation here is to understand separation ahead of flaps or elevons on swept wings, the flow phenomenon can be understood more readily by considering separation ahead of forward facing steps on swept wings. Reattachment occurs close to the step shoulder.[6,7] The length

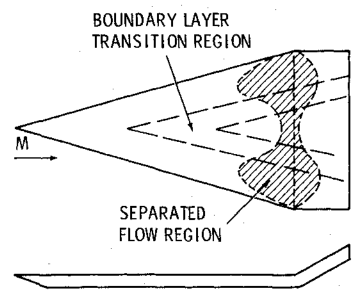

Fig. 1 "Dumbbell" shaped region of separated flow (Ref. 5).

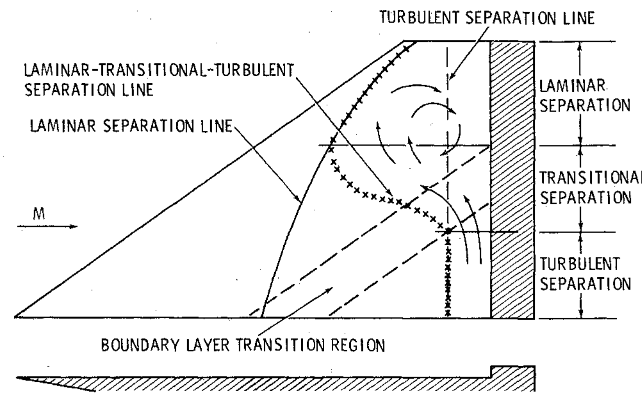

Fig. 2 Separation line shapes on a swept wing ahead of a forward facing step.

of separation ahead of the step can be estimated by using the step height h and assuming a straight dividing streamline. The angle of the dividing streamline α is estimated using oblique shock relation[8] for the average pressure level, $P = p_p/p_1$, on the plate surface in the separated flow region ahead of the step. Thus[8]

$$\ell = \frac{h}{\tan\alpha} = h\left[\frac{7M^2-5(P-1)}{5(P-1)}\right]\sqrt{\frac{6P+1}{7M^2-(6P+1)}} \tag{1}$$

where M is the undisturbed flow Mach number.

For two-dimensional separated flows ahead of steps, empirical expressions have been well established for the average pressure rise P for either laminar[9]

$$P = 1 + 1.22M^2\left[(M^2-1)Re\right]^{-1/4} \tag{2}$$

or turbulent[10]

$$P = \begin{cases} 1 + 2.24M^2 \Big/ \left[8 + (M-1)^2\right] & \text{for } M < 3.4 \\ 0.091M^2 - 0.05 + 6.37/M & \text{for } M > 3.4 \end{cases} \tag{3}$$

separation. For turbulent separation, the average pressure rise in the separated flow region can be estimated as a function of M alone [Eq. (3)]. Knowing M, and hence P from Eq. (3), the extent of turbulent separation can be estimated using Eq. (1). The separation line calculated for fully turbulent boundary-layer flows for a sample case is indicated by the dashed line in Fig. 2. For laminar separation, P is a function of both M and also the Reynolds number, Re, based on local flow conditions and the streamwise distance from the wing leading edge to the separation location. In the laminar case, Eqs. (1) and (2) must be solved iteratively to yield a separation locus such as that indicated by the solid curved line in Fig. 2.

If there are substantial regions of both laminar and turbulent flow on the wing surface, and the proposed strip

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*Staff Scientist, Research Department. Member AIAA.

†Aerospace Development Engineer.